Remark 2. The initial and boundary conditions are dictated by the structure of the differential equations, and the formulation of initial and boundary-value problems is the subject of a separate publication.

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UNSTEADY THREE-DIMENSIONAL LAMINAR BOUNDARY LAYER ON BLUNT BODIES WITH STRONG BLOWING

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One must investigate unsteady heat and mass transfer in flow of a compressible gas over blunt bodies with a permeable surface in order to solve many applied problems. In particular, these problems arise unavoidably and in general are time-dependent when gas is blown through a porous or perforated surface in order to form a gas curtain. Similar questions arise also in examining a number of chemical technology facilities in various regimes of operation.

For these reasons the literature has a number of papers in which both approximate analytical methods [1, 2] and numerical methods [3-5] have been used to study unsteady processes occurring in laminar planar or axisymmetric boundary layers in a compressible gas on a permeable surface. The influence of blowing (or suction) on the characteristics of the unsteady two-dimensional boundary layer was examined in [6, 7]. Unsteady heat transfer in the vicinity of a stagnation point with two radii of curvature was the subject of [8, 9], and the influence of strong blowing on the basic characteristics of steady flow in a three-dimensional laminar boundary layer was examined in [10-13].

This paper has obtained numerical and asymptotic solutions, over a wide range of variation of the governing parameters, of the equations of the unsteady three-dimensional laminar boundary layer on a permeable surface, including the case of strong blowing.

1. Statement of the Problem. We consider three-dimensional unsteady flow of a supersonic gas over blunt bodies with a permeable surface at large incident stream Reynolds number Re. We choose a nondegenerate curvilinear coordinate system (x^1, x^2, x^3) with origin at the stagnation point, and normally related to the wetted surface: x^3 = const is a family of surfaces parallel to the body surface ($x^3 = 0$), and x^1 and x^2 are curvilinear coordinates on the surface.

Later we shall also investigate bodies for which the longitudinal pressure gradient ∇p^* obtained by solving the equations describing inviscid flow over a given body is a quantity of order $O(\rho_\infty V_\infty^2/L)$. As is shown by asymptotic analysis of the unsteady three-dimensional Navier–Stokes equations for the case of hypersonic flow over bodies with blowing present and under the conditions

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$$\frac{\rho_w^* v_w^*}{\rho_\infty V_\infty} \leqslant O(1), \quad \lambda = \frac{\rho_w^* v_w^{*2}}{\rho_\infty V_\infty^2} \ll 1, \quad \mathrm{Re} = \frac{\rho_\infty V_\infty L}{\mu_0^*} \gg 1, \tag{1.1}$$

the equations of the unsteady three-dimensional laminar boundary layer give an asymptotically correct description of the flow in the layer near the body surface. In the coordinate system (x^1, x^2, x^3) these differ in appearance from the equations describing steady flow in a three-dimensional boundary layer, and given, e.g., in [13], only in respect of a number of unsteady terms: in the continuity equation one must add the term $\sqrt{g}\partial\rho/\partial t$, and in all the others replace the operator D by $D^* \equiv D + \partial/\partial t$. These equations are solved with the boundary conditions

$$x^3 \to \infty: u = u_e(x^1, x^2, t), \ w = w_e(x^1, x^2, t), \ T = T_e(x^1, x^2, t); \tag{1.2}$$

$$x^3 = 0: u = u_w(x^1, x^2, t), \ w = w_w(x^1, x^2, t), \ T = T_w(x^1, x^2, t), \tag{1.3}$$

$$\rho v = G(x^1, x^2, t), \quad \lim_{x^1, x^2 \to 0} \frac{u_w}{u_e} < \infty, \quad \lim_{x^1, x^2 \to 0} \frac{w_w}{w_e} < \infty.$$

For convenience of the numerical solution of the problem we convert in the original coordinate system and the boundary conditions to variables of the A. A. Dorodnitsyn type, with which we can resolve singularities at the stagnation point and also in the symmetry planes of the flow investigated:

$$\xi = x^1, \quad \eta = x^2, \quad \zeta = \frac{u_e}{\xi}\sqrt{\frac{g}{g_{11}}}\int_0^{x^3} \rho dx^3, \quad \tau = t,$$
$$u^* = \frac{u}{u_e} = \frac{\partial\varphi_1}{\partial\zeta}, \quad w^* = \frac{w}{w_e} = \frac{\partial\varphi_2}{\partial\zeta}, \quad \Theta = \frac{T}{T_e}, \quad l = \frac{\mu\rho}{\mu_e\rho_e}. \tag{1.4}$$

In the variables of Eqs. (1.4) the original system of equations and boundary conditions takes the form (we drop the superscript *)

$$(lu'_\zeta)'_\zeta = Du + \beta_1(u^2 - \Theta) + \beta_2(w^2 - \Theta) + \beta_3(uw - \Theta) + \gamma_1(u - \Theta),$$
$$(lw'_\zeta)'_\zeta = Dw + \beta_4(w^2 - \Theta) + \beta_5(u^2 - \Theta) + \beta_6(uw - \Theta) + \gamma_2(w - \Theta),$$
$$\left(\frac{l}{\sigma}\Theta'_\zeta\right)'_\zeta = D\Theta + \Theta(\beta_7 + \beta_8 u + \beta_9 w) - l\left[\alpha_6(u'_\zeta)^2 + \alpha_7 u'_\zeta w'_\zeta + \alpha_8(w'_\zeta)^2\right], \tag{1.5}$$
$$D \equiv \alpha_0\frac{\partial}{\partial\tau} + \alpha_1 u\frac{\partial}{\partial\xi} + \alpha_2 w\frac{\partial}{\partial\eta} - \left(\alpha_1\varphi'_{1\xi} + \alpha_2\varphi'_{2\eta} + \alpha_3\varphi_1 + \alpha_4\varphi_2 + \alpha_5\right)\frac{\partial}{\partial\zeta};$$

$$\zeta \to \infty: u = w = \Theta = 1; \tag{1.6}$$

$$\zeta = 0: u = u_w, \ w = w_w, \ \Theta = \Theta_w, \ (\xi\varphi_1)'_\xi + (\alpha_2\varphi_2)'_\eta = -\sqrt{g}\,G(\xi, \eta) \equiv F_w. \tag{1.7}$$

We shall not give expressions for the coefficients of Eqs. (1.5). We note only that they are known functions and depend on the time, the geometry of the wetted body and the pressure distribution along its surface.

2. Asymptotic Solution of the Problem with Strong Blowing. We consider the case when the blowing parameter, ordinarily used in laminar boundary layer theory [10-13], $f_w = \sqrt{\mathrm{Re}} \times \rho_w^* v_w^*/\rho_\infty V_\infty$ will be rather large. Then the problem becomes singular and we must solve it by the method of matched asymptotic expansions [14]. The boundary layer is divided into a blowing layer attached to the body, where the effects of molecular transport are insignificant in the first approximation, and a mixing layer where they play the major role.

Blowing Layer. The flow in the blowing layer is described in the first approximation by the system of equations

$$\frac{\partial}{\partial t}(\sqrt{g}\rho) + \frac{\partial}{\partial x}\left(\rho u\sqrt{\frac{g}{g_{11}}}\right) + \frac{\partial}{\partial y}\left(\rho w\sqrt{\frac{g}{g_{22}}}\right) + \frac{\partial}{\partial z}(\rho v\sqrt{g}) = 0,$$
$$\rho(Du + A_1u^2 + A_2w^2 + A_3uw) = A_4,$$
$$\rho(Dw + B_1u^2 + B_2w^2 + B_3uw) = B_4, \tag{2.1}$$
$$\rho DT = \frac{\gamma - 1}{\gamma}D^0 p, \quad x \equiv x^1, \quad y \equiv x^2, \quad z \equiv x^3,$$

which is solved with the initial conditions

$$t = 0: u = u^0(x, y, z), \ w = w^0(x, y, z), \ T = T^0(x, y, z), \ v = v^0(x, y, z) \tag{2.2}$$

(the superscript zero denotes the steady solutions of the system (2.1), given in [11, 12]). As boundary conditions for the system (2.1) we take conditions (1.3) on the body surface.

We consider the case when the pressure at the outer edge of the boundary layer and the gas temperature at the stagnation point are independent of time:

$$\frac{\partial p}{\partial t}=0,\quad \frac{\partial T_w}{\partial t}(0,0)=0. \tag{2.3}$$

Then the solution of the system (2.1), (2.2) and (1.3) in the vicinity of the stagnation point can be written in the quadratures:

$$\begin{gathered}
\left|\frac{(u(t,\tau)+1)(U(\tau)-1)}{(u(t,\tau)-1)(U(\tau)+1)}\right|=\exp[2(t-\tau g(\tau))],\\
\left|\frac{(w(t,\tau)+1)(W(\tau)-1)}{(w(t,\tau)-1)(W(\tau)+1)}\right|=\exp[2\alpha(t-\tau g(\tau))],\\
z(t,\tau)=\tau(g(\tau)-1)+\int\limits_{\tau g(\tau)}^{t}v\,dt,\\
v(t,\tau)=V(\tau)-\int\limits_{\tau(g(\tau)-1)}^{z}(u+\alpha w)\,dz,\\
U(\tau)=\begin{cases}u_w(\tau), & \tau\geqslant 0,\\ u^0(-\tau), & \tau<0,\end{cases}\quad W(\tau)=\begin{cases}w_w(\tau), & \tau\geqslant 0,\\ w^0(-\tau), & \tau<0,\end{cases}\\
V(\tau)=\begin{cases}v_w(\tau), & \tau\geqslant 0,\\ v^0(-\tau), & \tau<0,\end{cases}\quad \tau=\begin{cases}t^*, & \tau\geqslant 0,\\ -z^*, & \tau<0,\end{cases}\quad g(\tau)=\frac{1}{2}(\operatorname{sgn}(\tau)+1),\\
\alpha=\sqrt{\frac{p_{2y}}{p_{2x}}},\quad p_{2x}=-\frac{\gamma-1}{2\gamma}\rho_w^{-1}\frac{\partial^2 p}{\partial x^2}(0,0),\quad p_{2y}=-\frac{\gamma-1}{2\gamma}\rho_w^{-1}\frac{\partial^2 p}{\partial y^2}(0,0).
\end{gathered} \tag{2.4}$$

Here t^*, z^* are the coordinates t and z of the exit of the characteristic of the system (2.1) in the (t, z) plane with the coordinate lines z = 0 and t = 0, respectively; $u_1=p_{2x}^{-1/2}u_x'$; $w_1=p_{2y}^{-1/2}w_y'$; $z_1=p_{2x}^{1/2}z$; $t_1=p_{2x}^{1/2}t$; and the subscript 1 is omitted. The values of all the quantities on the dividing stream line are obtained from Eqs. (2.4), if there we put $\tau=-z^0$ (z^0 is the coordinate z of the dividing stream line at the stagnation point in the steady solution).

In the general case the solution for the profiles of velocity and temperature in the blowing layer can be found either numerically or in the form of series in the normal coordinate z.

On the body surface we have asymptotic formulas for the components of the friction stress and the heat flux:

$$\begin{gathered}
\tau_x\equiv\mu\frac{\partial u}{\partial z}\Big|_{z=0}=\frac{\mu}{(\rho v)_w}\left[A_4-(A_1u_w^2+A_2w_w^2+A_3u_ww_w)\rho_w-\rho_w\left(\frac{\partial u_w}{\partial t}+\frac{u_w}{\sqrt{g_{11}}}\frac{\partial u_w}{\partial x}+\frac{w_w}{\sqrt{g_{22}}}\frac{\partial u_w}{\partial y}\right)\right],\\
\tau_y\equiv\mu\frac{\partial w}{\partial z}\Big|_{z=0}=\frac{\mu}{(\rho v)_w}\left[B_4-\rho_w\left(B_1u_w^2+B_2w_w^2+B_3u_ww_w+\frac{\partial w_w}{\partial t}+\frac{u_w}{\sqrt{g_{11}}}\frac{\partial w_w}{\partial x}+\frac{w_w}{\sqrt{g_{22}}}\frac{\partial w_w}{\partial y}\right)\right],\\
q\equiv\lambda\frac{\partial T}{\partial z}\Big|_{z=0}=-\frac{\lambda}{c_p(\rho v)_w}\left[c_p\rho_w\left(\frac{\partial T_w}{\partial t}+\frac{u_w}{\sqrt{g_{11}}}\frac{\partial T_w}{\partial x}+\frac{w_w}{\sqrt{g_{22}}}\frac{\partial T_w}{\partial y}\right)-\frac{\partial p}{\partial t}-\frac{u_w}{\sqrt{g_{11}}}\frac{\partial p}{\partial x}-\frac{w_w}{\sqrt{g_{22}}}\frac{\partial p}{\partial y}\right].
\end{gathered}$$

Mixing Layer. In the vicinity of the dividing stream line $z=z^*(x,y,t)$ in the layer where the blown gas mixes with the oncoming stream Eqs. (2.1) becomes unsuitable. It can be shown that the equations governing the mixing layer structure coincide in form with the original equations of the unsteady three-dimensional boundary layer, if there we replace v by $V\equiv v-D^0z^*$. The boundary conditions here are the following:

$$\begin{gathered}
u\to u_e,\ w\to w_e,\ T\to T_e\ \text{for}\ z\to\infty,\\
u\to u_-,\ w\to w_-,\ T\to T_-\ \text{for}\ z\to-\infty,\\
V=0\quad \text{for}\ z=0,
\end{gathered}$$

and $u_-(x,y,t)$, $w_-(x,y,t)$, $T_-(x,y,t)$ are determined by solving the outer problem in the blowing layer on the dividing stream line. Here, as can be seen by analyzing the solution of the outer problem, if conditions (2.3) hold, then in the first approximation the structure of the mixing layer in the variables of Eqs. (1.4) does not depend on time and coincides with the steady-state solution obtained in [11, 15].

3. Numerical Solution of the Problem. Discussion of the Calculated Results. The system of equations (1.5)-(1.7) was solved numerically on a computer. We used an implicit difference scheme of fourth order accuracy in the coordinate ζ, a generalization of the scheme [16] for three-dimensional unsteady flow in the boundary layer. We considered flow at zero angle of attack over an elliptic paraboloid whose surface in a rectangular coordinate system (y^1, y^2, y^3) has an equation of the form

$$y^3 = 0{,}5[(y^1)^2 + k^2(y^2)^2], \tag{3.1}$$

where k is the ratio of the principal radii of curvature of the body at the stagnation point. The incident flow was considered hypersonic and steady-state, and the pressure at the outer edge of the boundary layer was determined by the Newtonian formula. The coordinates (x^1, x^2) on the body surface were chosen analogously to [13], and the governing parameters of the problem were varied in the range

$$0.01 \leqslant k \leqslant 1, \quad 0.1 \leqslant \Theta_w \leqslant 0.5, \quad 0.5 \leqslant \omega \leqslant 1.0, \quad \sigma = 0.71, \quad 0 \leqslant -F_w \leqslant 5. \tag{3.2}$$

Here the mass flow rate of gas through the surface F_w and its temperature Θ_w are piecewise smooth functions of ξ, η, τ, and allow discontinuities of the first kind.

During the solution we found the profiles of velocity and temperature across the boundary layer, and also the coefficients of friction and heat transfer

$$c_\xi \equiv \mu \frac{\partial u}{\partial \zeta}, \quad c_\eta \equiv \mu \frac{\partial w}{\partial \zeta}, \quad c_q \equiv \lambda \frac{\partial \Theta}{\partial \zeta}. \tag{3.3}$$

We now consider flow in the vicinity of the stagnation point with two radii of curvature. As an example we take the continuous and discontinuous dependences of F_w, Θ_w on time:

$$F_w = F_w^0 - \sin^2\tau, \quad F_w = \begin{cases} a_1 & (\tau = 0), \\ a_2 & (\tau > 0); \end{cases} \tag{3.4a,b}$$

$$\Theta_w = \begin{cases} b_1 & (\tau = 0), \\ b_1 + b_2\tau & (0 < \tau \leqslant \tau_0), \\ b_1 + b_2\tau_0 & (\tau > \tau_0), \end{cases} \tag{3.5a}$$

$$\Theta_w = \begin{cases} c_1 & (\tau = 0), \\ c_2 & (\tau > 0) \end{cases} \tag{3.5b}$$

(Here the a_i, b_i, and c_i are constants.)

Some results of the calculations are shown in Figs. 1-3. Figure 1 shows the profiles of u (lines 1, 2, and 4) and w (lines 3, 5, and 6) across the boundary layer for τ = 0.03; 3.3; 10.3 (lines 4 and 6; 2 and 5; and 1 and 3, respectively) for k = 0.5, $F_w = -10$, $\omega = 0.5$, $\gamma = 1.4$ and with Θ_w given by the law of Eq. (3.5a) ($b_1 = 0.1$, $b_2 = 1$, $\tau_0 = 0.15$); the broken lines show the asymptotic solution of the problem in the two layers,

It can be seen that although for $\tau > \tau_0$ the body surface temperature Θ_w does not depend on time, the structure of the layer of blown gases depends for quite a long time on the time. Here, as follows from the asymptotic solution, inside the blowing layer local extrema are formed in the profiles of velocity and temperature. On the whole by analyzing the solutions obtained we can draw the conclusion that, as in the steady-state case [13], the asymptotic solution has good accuracy for $-F_w \gtrsim$ 3-5.

The calculations made allow a number of interesting flow laws to be identified. Firstly one should note that the absolute values of the coefficients of friction and heat transfer on the body surface depend strongly on the governing parameters of the problem. For example, with the boundary conditions given in the form of Eqs. (3.4a) and (3.5a) the variation of c_ξ, c_q with increase of k from 0.01 to 1 was 40-50%. However, the relative values of the components of the friction stress and the heat flux, referenced to their steady-state values

$$c_\xi^0 = \frac{c_\xi(\tau)}{c_\xi(0)}, \quad c_\eta^0 = \frac{c_\eta(\tau)}{c_\eta(0)}, \quad c_q^0 = \frac{c_q(\tau)}{c_q(0)}, \tag{3.6}$$

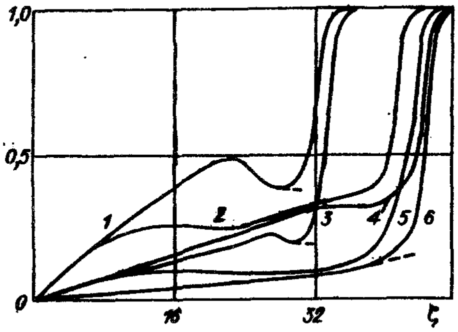

Fig. 1

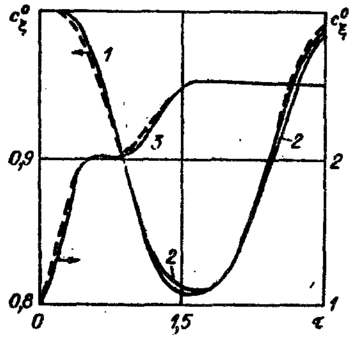

Fig. 2

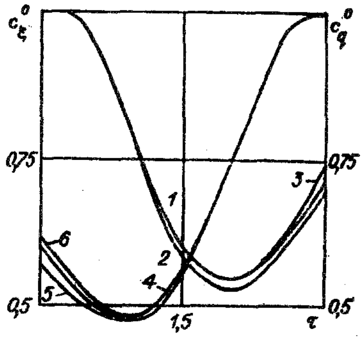

Fig. 3

are appreciably more conservative, and depend only slightly on a number of the governing parameters of the problem.

Firstly, as shown by the calculations, c_ξ^0, c_η^0, c_q^0 are practically independent of the geometrical parameter k. For example, with F_w given according to the law of Eq. (3.4a) the value of c_q^0 found with k = 0.01 differs from the value of c_q^0 calculated with k = 0.75, by 1.5-2%. The corresponding variation of c_ξ^0, c_η^0 was 2-4%. Secondly, these characteristics depend weakly on the parameters γ, ω.

As an example subject to the above flow laws we can take Fig. 2, which shows c_ξ^0 as a function of time for various laws for $F_w(\tau)$ and $\Theta_w(\tau)$. Here lines 1 and 2 correspond to the boundary conditions (3.4a) ($F_w^0 = -4$); line 3 corresponds to conditions (3.4a) ($F_w^0 = -4$); (3.5a) ($b_1 = 0.1$, $b_2 = 0.25$, $\tau_0 = 0.6$); the broken lines correspond to the asymptotic solution with strong blowing; line 1 for k = 0.01, $\Theta_w = 0.25$, γ = 1.4; line 2 for k = 0.5, $\Theta_w = 0.1$, γ = 1.1; line 3 for k = 0.75, γ = 1.2. Figure 3 shows the analogous dependences of c_ξ^0 and c_q^0 (lines 1-3 and 4-6, respectively) for the boundary condition (3.4a) ($F_w^0 = -0.5$) and steady-state surface temperature. Here curves 1 and 4 are for k = 0.5, $\Theta_w = 0.1$, γ = 1.4; curves 2 and 5 for k = 1.0, $\Theta_w = 0.25$, γ = 1.2; curves 3 and 6 for k = 0.1, $\Theta_w = 0.2$, γ = 1.1.

Starting from the noted weak dependence of c_ξ^0, c_η^0, c_q^0 on the ratio of the principal radii of curvature of the body at the stagnation point k, we can suggest the following formula for calculating the absolute unsteady values of the components of the friction stress and the heat flux on the body surface in the vicinity of the stagnation point:

$$c_\xi(k,\tau) = c_\xi^*(k)\, c_\xi^0(1,\tau), \quad c_\eta(k,\tau) = c_\eta^*(k)\, c_\eta^0(1,\tau), \quad c_q(k,\tau) = c_q^*(k)\, c_q^0(1,\tau), \tag{3.7}$$

where $c_\xi^*(k)$, $c_\eta^*(k)$, $c_q^*(k)$ are determined from the steady-state solution of the problem and can be calculated from analytical formulas [13]; and $c_\xi^0(1,\tau)$, $c_\eta^0(1,\tau)$, $c_q^0(1,\tau)$ are the relative values of the components of friction stress and heat flux calculated for flow over an axisymmetric body. From Eqs. (3.7) we can calculate c_ξ, c_η, c_q for arbitrary k, knowing only the corresponding solutions for the axisymmetric case. Comparison of Eqs. (3.7) with the numerical solution over a wide range of variation of k, γ and the coefficients in conditions (3.4) and (3.5) has shown that their maximum difference does not exceed 7-8%.

We consider the flow in the vicinity of the plane of symmetry $y^2 = 0$. The surface temperature was considered to be steady-state, and the blowing parameter F_w as a function (including also a discontinuous function) of time and the coordinate $\xi = y^1$. During the calculation, in addition to the profiles of velocity and temperature we also determined the relative coefficients

$$\tau_\xi^0 = \frac{c_\xi(\xi,\tau)}{c_\xi(0,0)}, \quad \tau_\eta^0 = \frac{c_\eta(\xi,\tau)}{c_\eta(0,0)}, \quad q^0 = \frac{c_q(\xi,\tau)}{c_q(0,0)}, \tag{3.8}$$

for which the values at the stagnation point coincide with the corresponding values determined from Eq. (3.6). Some typical examples of the distribution of q^0 for k = 0.25, ω = 0.5, γ = 1.4 and other methods of assigning $F_w = F_w(\xi,\tau)$ and Θ_w are shown in Figs. 4 and 5: Fig. 4 shows the continuous dependence of F_w on time in the form of Eq. (3.4a) ($F_w^0 = -0.5$) for $\Theta_w = 0.1$; and Fig. 5 shows the calculation when $\Theta_w = 0.25$, $F_w = -0.5$ (for $\tau < 0.45$ and $0.05 \le \xi \le 0.25$) and $F_w = 0$ (in the remaining cases).

The comparisons show that the presence of a discontinuity in the vectorial velocity of the blown gas in both time and space has a strong influence on the character of τ_ξ^0, τ_η^0, q^0.

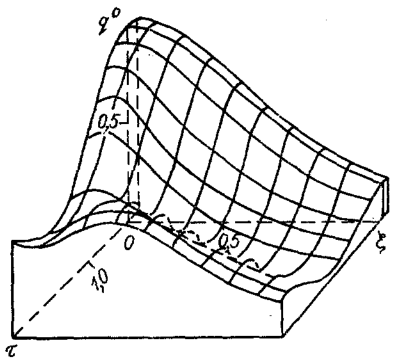

Fig. 4

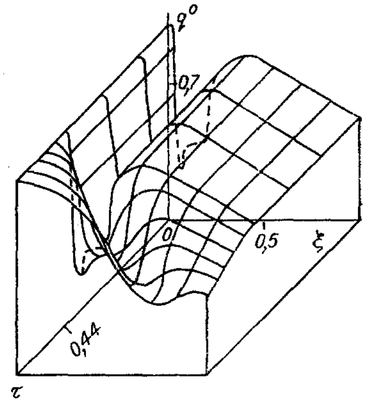

Fig. 5

However, from the calculations performed, on the whole one can conclude that the influence of the discontinuious character of the dependence of the boundary conditions on time and the spatial coordinates is localized to a great degree in the vicinity of the affected points or lines of discontinuity. An analogous result for steady flows in the three-dimensional boundary layer in the presence of a partially permeable section of the surface was obtained in [17].

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DISTRIBUTED INJECTION OF A GAS INTO A HYPERSONIC FLOW

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Distributed surface injection of a gas is used to reduce heat flows to the surface of aircraft traveling at high supersonic velocities. The injection changes the effective form of the surface and can therefore be used to create aerodynamic forces and moments. The latter case is characterized by velocities normal to the injection surface which are an order of magnitude greater than the vertical velocity in the boundary layer on an impermeable surface. Flow regimes with intensive injection have been studied in several investigations, a survey of which is offered in [1]. At the same time, for the goal of protection from heating, it is optimum if the flow rate of the injected gas is comparable to the flow rate in the boundary layer on an impermeable surface, since the intensity of the injection ensures a reduction in heat flux in the dominant term. In this case, flow near the permeable surface is described by a system of boundary-layer equations. Hypersonic flows are characterized by the highest heat fluxes, and this is particularly true for the regime of strong hypersonic interaction.

Studies of flows for this regime have been limited mainly to examining problems with boundary conditions, which provide for a reduction in the system of boundary-layer equations to a system of ordinary differential equations [2]. At the same time, the distribution of injection rate realized in practice makes it necessary to solve problems which are not self-similar. An example of the solution of such problems is given in the present study.

There is yet one more circumstance which makes the study of flows with injection particularly important. In classical boundary-layer theory, there are two types of singularities in the solution. These singularities are connected with the vanishing of skin friction and with alteration of the structure of the flow. In the first case, friction decreases to zero and a region of reverse currents is formed (the boundary layer separates) due to an unfavorable pressure gradient. In the second case, distributed injection causes friction to vanish and a region of inviscid boundary flow to form (the boundary layer is detached). The structure of flow in the boundary layer is determined by diffusion and convection associated with vorticity. At large Reynolds numbers, the distance over which the vorticity diffuses from the solid surface is much less than the distance over which the vorticity is transported along the surface by convection [3]. Stagnation of the fluid under the influence of an unfavorable pressure gradient leads to development of the convective mechanism of vorticity transport from the surface and to restructuring of the flow in the boundary layer. Such convection also develops as a result of surface injection. The solutions of the system of boundary-layer equations near points of zero friction were described mathematically in [4, 5]. Analysis of these solutions showed that a large unfavorable pressure gradient, induced by the displacement thickness in the external flow, develops in the vicinity of points of zero skin friction. By allowing for the interaction of the boundary-layer flow with the external flow, it was possible for investigators to obtain a smooth solution which passed through the separation point in supersonic [6, 7] and subsonic [8] flows. It later turned out that allowing for an induced pressure gradient in a composite system of boundary-layer equations makes it possible to also eliminate the singularity for the solution which describes flow with distributed injection [9]. The solution obtained in [9] corresponded to the regime of weak interaction, and the induced pressure gradient began to have an appreciable effect only after skin friction was reduced to nearly zero. The strong interaction regime is characterized by the fact that the boundary-layer flow and the inviscid external flow influence each other along the entire surface of the body. Thus, if it exists at all, the phenomenon of boundary-layer detachment should have several features which will distinguish it from the analogous phenomenon in the weak interaction regime. It is the analysis of these features which is the focus of this article.

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